Interface matching method for solving surface plasmon modes with damping in plasmonic crystals

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The author proposes an interface matching method for solving surface plasmon modes with damping in plasmonic crystals. The damping constant is considered a crucial parameter instead of a small perturbation to the undamped system. The damping effect is manifest on the complex nature of the eigenfrequency as well as on the eigenfield. For periodic layered structures, the decay factors of the two fundamental modes asymptotically approach $\gamma/2$ in the large-wave-number limit. For two-dimensional plasmonic crystals, the decay factors of surface plasmon modes are gathered around and bounded by $\gamma/2$.

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I. INTRODUCTION

Surface plasmons are electromagnetic waves that propagate along the surface of a conductor [1]. These waves come from collective excitations of the electric charges, coupled with the external electromagnetic fields. In addition to metals, surface plasmons occur as well in semiconductors [2,3], semimetals [4], and metamaterials [5], Surface plasmons may therefore spread a wide range of frequency from the optical (hundreds of terahertz) to infrared (terahertz), and even microwave (gigahertz) regimes. The most distinguishing feature of surface plasmons is the exponential decay of field amplitude away from the surface. This property is also referred to as evanescent, near-field, bound, and nonradiative. Making use of the property that the fields are largely confined within a very narrow region with strong enhancement, surface plasmons are being explored in optical data storage [6,7], biosensing [8,9], light generation [10,11], photonic circuits [12,13], solar cells [14,15], and so forth. With the progress of nanofabrication technologies and microscopic measurement techniques [16,17], even more applications are proposed.

The problem of solving surface plasmon modes involves dealing with a nonlinear eigenvalue problem, as the frequency is considered an unknown [18]. The interface matching method was proposed to solve the problem in an accurate and efficient manner, without resorting to nonlinear techniques. On the one hand, the original eigensystem is reformatted as a linear equation that can be solved by standard algorithms. On the other hand, the highly localized feature associated with surface plasmons is resolved through the use of the interface condition. This approach has been successfully applied to solve surface plasmon modes in plasmonic crystals [19], polaritonic crystals [20], negative index crystals [21], plasmonic hole waveguides [22], and plasmonic split-ring structures [23].

In this Brief Report, the author extends the interface matching method to be applicable to surface plasmon modes with damping. Instead of considering the damping effect as a perturbation to the undamped system, where the damping constant γ is a small number [19], the present approach regards γ as a crucial parameter that can be specified according to real situations. By use of the Drude model, the underlying problem is reformulated as a cubic eigensystem, which in turn is recast into a linear equation and solved by standard algorithms. The damping effect is manifest on the complex nature of the eigenfrequency as well as the eigenfield. On the one hand, the complex eigenfrequency exhibits a decay factor of the field amplitude in time, when the wave vector is considered real. On the other hand, the complex eigenfield indicates a phase lag of the field oscillation, as compared to the case without damping. These features are reported in periodic layered structures, where the decay factors of the two fundamental branches (usually termed the acoustical and optical modes) asymptotically approach $\gamma/2$ as the wave number goes to infinity. For two-dimensional plasmonic crystals, the decay factors associated with surface plasma modes are intensively gathered around and bounded by $\gamma/2$.

II. INTERFACE MATCHING METHOD

The basic idea of the interface matching method is to deal with the wave equation strictly in the interior of two different media, so that the equation in either region can be rearranged as a standard equation by employing a suitable dielectric model. The two equations are then connected by matching the boundary condition at the interface. A general formulation is utilized to combine together the equations in the interior as well as at the interface, and to give rise to a generalized eigensystem that can be efficiently solved by standard algorithms.

A. Basic equations

Based on Maxwell's equations, the time-harmonic electromagnetic fields (with time dependence $e^{-i\omega t}$) in a linear, isotropic, and nonmagnetic medium satisfy the following wave equations [24]:

$$\frac{1}{\varepsilon}\nabla\times\nabla\times\mathbf{E}=\left(\frac{\omega}{c}\right)^2\mathbf{E}, \tag{1}$$

$$\nabla\times\frac{1}{\varepsilon}\nabla\times\mathbf{H}=\left(\frac{\omega}{c}\right)^2\mathbf{H}. \tag{2}$$

Consider two-dimensional structures with periodicity in the xy plane and the geometry constant along the z axis. The

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problem can be classified into transverse magnetic (TM) and transverse electric (TE) polarizations with respect to the z axis; that is, TM refers to the case with $H_z=0$ and TE refers to $E_z=0$. Either case can be described in terms of the z component as

$$-\nabla^2 E_z = \varepsilon\left(\frac{\omega}{c}\right)^2 E_z, \tag{3}$$

$$-\boldsymbol{\nabla}\cdot\left(\frac{1}{\varepsilon}\boldsymbol{\nabla} H_z\right) = \left(\frac{\omega}{c}\right)^2 H_z, \tag{4}$$

for TM and TE polarizations, respectively. Equations (3) and (4) are considered eigenvalue problems regarding the frequency as an unknown. For a frequency-dependent dielectric function $\varepsilon=\varepsilon(\omega)$, the differential operator in Eq. (4) depends on the problem to be solved and the corresponding eigensystem becomes nonlinear in frequency [18]. In addition, either Eq. (3) or (4) contains two unknowns, ω and E_z (or H_z) and there needs to be an additional condition for the problem to be complete. For periodic structures, the periodicity along the lattice serves as a physical constraint that the fields are subject to Bloch's condition: $\phi(\mathbf{r}+\mathbf{a}_i)=e^{i\mathbf{k}\cdot\mathbf{a}_i}\phi(\mathbf{r})$, where ϕ is either E_z or H_z, $\mathbf{k}=k_x\hat{\mathbf{x}}+k_y\hat{\mathbf{y}}$ is the Bloch wave vector, and $\mathbf{a}_i$ $(i=1,2)$ is the lattice translation vector in the xy plane. The problem for the periodic structure with infinite extent is solved within one unit cell.

Let the structure consist of two different media: plasmonic material and a surrounding dielectric. Strictly in the interior of the dielectric or metal where *the interface is excluded*, Eq. (3) or (4) is simplified to

$$-\nabla^2\phi = \varepsilon\left(\frac{\omega}{c}\right)^2\phi. \tag{5}$$

To account for the damping effect, the Drude model $\varepsilon=1-\omega_p^2/(\omega^2+i\omega\gamma)$ is used for the dielectric function of the plasmonic material, where ω_p is the bulk plasma frequency and γ is the damping constant. Let ε_d be the dielectric constant of the surrounding material; Eq. (5) can be rearranged as

$$(\Omega^2\varepsilon_d+\nabla^2)\phi=0, \tag{6}$$

$$[\Omega^3+\Omega^2(i\Gamma)+\Omega(\nabla^2-\Omega_p^2)+i\Gamma\nabla^2]\phi=0, \tag{7}$$

for ϕ in the dielectric and plasmonic materials, respectively, where $\Omega\equiv\omega/c$, $\Omega_p\equiv\omega_p/c$, and $\Gamma\equiv\gamma/c$. The above two equations in the interiors are connected by matching the boundary condition at the interface between the two media. According to Faraday's law, $\boldsymbol{\nabla}\times\mathbf{E}=i\omega\mathbf{H}$, the tangential magnetic field for TM polarization is given as $\hat{\mathbf{n}}\times\mathbf{H}=(i/\omega)(\partial E_z/\partial n)\hat{\mathbf{z}}$, where $\hat{\mathbf{n}}$ is the unit vector normal to the interface. Likewise, according to Ampere-Maxwell's law $\boldsymbol{\nabla}\times\mathbf{H}=-i\omega\varepsilon\mathbf{E}$, the tangential electric field for TE polarization is given as $\hat{\mathbf{n}}\times\mathbf{E}=-(i/\omega\varepsilon)(\partial H_z/\partial n)\hat{\mathbf{z}}$. Continuity of the tangential field components at the interface thus requires

$$\left(\frac{\partial E_z}{\partial n}\right)_S=0, \quad \left(\frac{1}{\varepsilon}\frac{\partial H_z}{\partial n}\right)_S=0, \tag{8}$$

where $(\cdots)_S$ denotes the jump across the interface S. Using the Drude model for the plasmonic material, one has

$$\left.\frac{\partial\phi}{\partial n}\right|_+ = \left.\frac{\partial\phi}{\partial n}\right|_-, \tag{9}$$

$$\left.\frac{1}{\varepsilon_d}\frac{\partial\phi}{\partial n}\right|_+ = \left.\frac{\Omega^2+i\Omega\Gamma}{\Omega^2+i\Omega\Gamma-\Omega_p^2}\frac{\partial\phi}{\partial n}\right|_-, \tag{10}$$

for the TM and TE polarizations, respectively, where + refers to the dielectric side and − to the metal side. It is noted that Eq. (9) is always satisfied once a smooth solution of Eq. (5) is obtained. The matching of the interface is therefore not necessary for TM polarization. For TE polarization, Eq. (10) depends on ω and needs an explicit treatment in the solution procedure, which will be addressed further in the next section. It is also noted that surface plasmons exist only for TE polarization, where the interface condition plays an important role in the characteristics of surface plasmons [22].

B. Eigensystems

Let $\phi_{i,j}$ $(i,j=0,\ldots,N)$ be the discrete value of ϕ at the point (ih,jh) in the unit cell, where $h\equiv a/N$ is the uniform spacing of discretization and a is the lattice constant. Strictly in the interior of the dielectric or plasmonic material, the central difference scheme is used to discretize Eqs. (6) and (7), giving rise to

$$(\Omega^2\varepsilon_d-L)\phi_{i,j}=0, \tag{11}$$

$$[\Omega^3+\Omega^2(i\Gamma)-\Omega(L+\Omega_p^2)-i\Gamma L]\phi_{i,j}=0, \tag{12}$$

where $L\phi_{i,j}=(1/h^2)(\Delta_x^+ +\Delta_x^- +\Delta_y^+ +\Delta_y^-)\phi_{i,j}$, with $\Delta_x^\pm\phi_{i,j}=\phi_{i,j}-\phi_{i\pm1,j}$ and $\Delta_y^\pm\phi_{i,j}=\phi_{i,j}-\phi_{i,j\pm1}$. At the interface between the dielectric and plasmonic materials, the one-sided difference scheme is utilized to discretize Eq. (10) on either side of the interface, and yields

$$[\Omega^2 B+\Omega(i\Gamma B)-A]\phi_{i,j}=0, \tag{13}$$

where

$$B\phi_{i,j}=\begin{cases}(\Delta_h^-+\varepsilon_d\Delta_h^+)\phi_{i,j} & \text{for } \hat{\mathbf{n}}=(-1,0),\\ (\varepsilon_d\Delta_h^-+\Delta_h^+)\phi_{i,j} & \text{for } \hat{\mathbf{n}}=(+1,0),\\ (\Delta_v^-+\varepsilon_d\Delta_v^+)\phi_{i,j} & \text{for } \hat{\mathbf{n}}=(0,-1),\\ (\varepsilon_d\Delta_v^-+\Delta_v^+)\phi_{i,j} & \text{for } \hat{\mathbf{n}}=(0,+1),\end{cases} \tag{14}$$

and

$$A\phi_{i,j}=\begin{cases}\Omega_p^2\Delta_h^\pm\phi_{i,j} & \text{for } \hat{\mathbf{n}}=(\pm1,0),\\ \Omega_p^2\Delta_v^\pm\phi_{i,j} & \text{for } \hat{\mathbf{n}}=(0,\pm1).\end{cases} \tag{15}$$

Note that $B\phi_{i,j}$ and $A\phi_{i,j}$ depend on the unit normal vector $\hat{\mathbf{n}}$ of the interface, pointing from inside the metal to outside. Near the surface plasmon resonance, the solutions will be highly localized at the interface and this feature is resolved by matching the normal derivatives from either side of the interface [cf. Eq. (10)].

The discretized equation for ϕ_i strictly in the interior [Eqs. (11) and (12)] as well as at the interface [Eq. (13)] are arranged together to yield a cubic system equation,

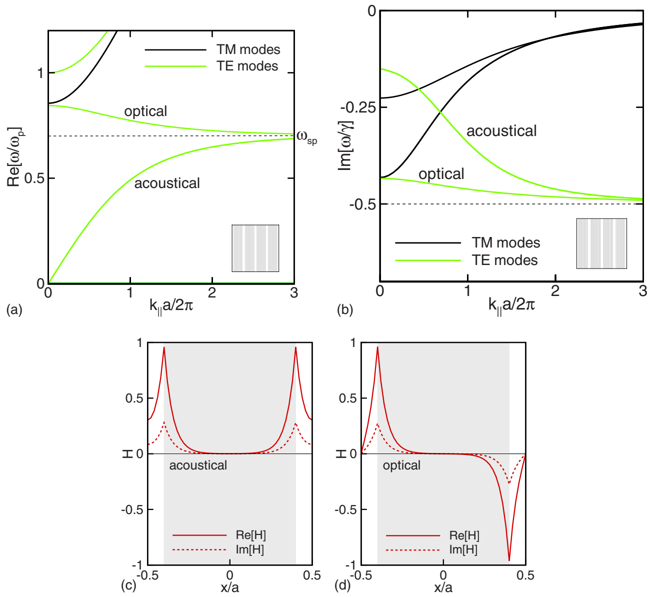

FIG. 1. (Color online) Dispersion relation and mode patterns for a periodic metal layered structure with $f=0.8$ based on the Drude model with $\omega_p a/2\pi c=1$ and $\gamma/\omega_p=0.2$ at $k_\perp=0$. (a) Real and (b) imaginary frequency branches; (c) acoustical and (d) optical mode at $k_\| a/2\pi=3$. The shaded area corresponds to the metal region. $k_\perp$ and $k_\|$ are the wave number components perpendicular and parallel to the metal surface, respectively.

$$(\Omega^3\mathbf{B}-\Omega^2\mathbf{D}-\Omega\mathbf{A}-\mathbf{E})\mathbf{x}=\mathbf{0}, \tag{16}$$

where $\mathbf{B}$, $\mathbf{D}$, $\mathbf{A}$, and $\mathbf{E}$ are square matrices and $\mathbf{x}$ is the eigenvector consisting of all $\phi_{i,j}$. The eigensystem (16) is made complete by incorporating the Bloch condition at the unit cell boundary, in which the field values at the left and right sides are related by $\phi_{N,j}=\phi_{0,j}e^{i\mathbf{k}\cdot\mathbf{a}_1}$, and $\phi_{i,N}=\phi_{i,0}e^{i\mathbf{k}\cdot\mathbf{a}_2}$. Introducing two auxiliary vectors $\mathbf{y}=\Omega\mathbf{x}$ and $\mathbf{z}=\Omega^2\mathbf{x}$, Eq. (16) can be recast into

$$\begin{bmatrix}\mathbf{0}&\mathbf{I}&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\mathbf{I}\\\mathbf{E}&\mathbf{A}&\mathbf{D}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\\mathbf{y}\\\mathbf{z}\end{bmatrix}=\Omega\begin{bmatrix}\mathbf{I}&\mathbf{0}&\mathbf{0}\\\mathbf{0}&\mathbf{I}&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\mathbf{B}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\\mathbf{y}\\\mathbf{z}\end{bmatrix}, \tag{17}$$

which is a generalized linear eigensystem: $\mathbf{A}'\mathbf{x}'=\Omega\mathbf{B}'\mathbf{x}'$. Note that the entries of $\mathbf{A}'$ and $\mathbf{B}'$ are all independent of Ω and the overall system can be solved by standard algorithms [25,26].

III. RESULTS AND DISCUSSION

In the presence of damping, the dispersion characteristics of surface plasmons alter in two aspects. First, the dielectric function ε is complex and solutions of the corresponding eigensystem are no longer real. Figure 1 shows the dispersion relation for a periodic metal layered structure with the fraction $f=0.8$ based on the Drude model with $\omega_p a/2\pi c=1$, $\gamma/\omega_p=0.2$, and $\varepsilon_d=1$. The real and imaginary parts of the eigenfrequencies are represented in Figs. 1(a) and 1(b), respectively. The effect of damping is manifest on the two fundamental TE frequency branches. The real eigenfrequency $\mathrm{Re}(\omega)$ is slightly attenuated, while the imaginary eigenfrequency $\mathrm{Im}(\omega)$ grows in proportion to γ. In addition, the optical (higher) branch has a more negative $\mathrm{Im}(\omega)$ than the acoustical (lower) branch at $k_\|=0$. As $k_\|$ increases, $\mathrm{Im}(\omega)$ asymptotically approaches $-\gamma/2$ for both branches. This feature can be characterized by the dispersion for a planar metal surface, given by [27,28]

$$k_\| = \frac{\omega}{c}\sqrt{\frac{\varepsilon_d\varepsilon_m(\omega)}{\varepsilon_d+\varepsilon_m(\omega)}}. \tag{18}$$

As $\varepsilon_d+\varepsilon_m(\omega)$ approaches zero, $k_\|$ goes to infinity and the asymptotic frequency is identified as the surface plasma frequency ω_{sp}. For systems without damping, that is, $\varepsilon=1-\omega_p^2/\omega^2$, one has $\omega_{\mathrm{sp}}=\omega_p/\sqrt{\varepsilon_d+1}$. This asymptotic behavior holds for other structures such as metal films [29], periodic metal layers [19], plasmonic crystals [19], and plasmonic hole waveguides [22]. For systems with damping, that is, $\varepsilon=1-\omega_p^2/(\omega^2+i\omega\gamma)$, the dispersion relation (18) becomes

$$\varepsilon_d\omega^4+i\gamma\varepsilon_d\omega^3-[\varepsilon_d\omega_p^2+(\varepsilon_d+1)k_\|^2c^2]\omega^2 - i\gamma(\varepsilon_d+1)k_\|^2c^2\omega+k_\|^2c^2\omega_p^2=0. \tag{19}$$

In the limit as $k_\|\to\infty$, Eq. (19) is simplified to $\omega^2+i\gamma\omega-\omega_p^2/(\varepsilon_d+1)=0$ and the asymptotic solution is given by

$$\omega_{\mathrm{sp}}=\sqrt{\frac{\omega_p^2}{\varepsilon_d+1}-\frac{\gamma^2}{4}}-\frac{i\gamma}{2}, \tag{20}$$

which is regarded as the surface plasma frequency in the presence of damping. Compared to the case for $\gamma=0$, the real part of ω_{sp} is slightly reduced and the imaginary part is equal to $-\gamma/2$. This feature is consistent with the result shown in Fig. 1(b). In another aspect, the presence of damping gives rise to complex eigenfields, as shown in Figs. 1(c) and 1(d) for the acoustical and optical modes, respectively, at $k_\| a/2\pi=3$. The imaginary part of the magnetic field is *in phase* with the real part, and is proportional to the damping constant. In view of the time dependence $e^{-i\omega t}$ used in the Drude model, the presence of damping results in a *phase lag* of the eigenfield. This feature consists with the mechanism of dissipation during the oscillation of fields [30].

Figure 2 shows the dispersion relation for a periodic array of square metal columns of width $0.6a$ with $\omega_p a/2\pi c=1$ and $\gamma/\omega_p=0.2$. The real parts of the frequency branches [Fig. 2(a)] retain the typical features of plasmonic crystals [19]. A large number of TE branches gather around the surface plasma frequency ω_{sp}, given by Eq. (20). As the frequency gets closer to ω_{sp}, more branches are observed. Due to the strong coupling of photons with electrons, these branches become dispersionless, or insensitive to the change of wave vector. They appear as flattened bands located within a rather small bandwidth. The effect of damping is manifest on the imaginary parts of TE branches [Fig. 2(b)], which basically grow in proportion to γ. As the real frequency branches come close to ω_{sp}, the imaginary parts of eigenfrequencies tend to approach and be bounded by $-\gamma/2$. This feature is consistent with that for the layered structure shown in Fig. 1. A typical surface plasmon mode pattern at the point Γ is shown in Figs. 2(c) and 2(d), respectively. The fields are

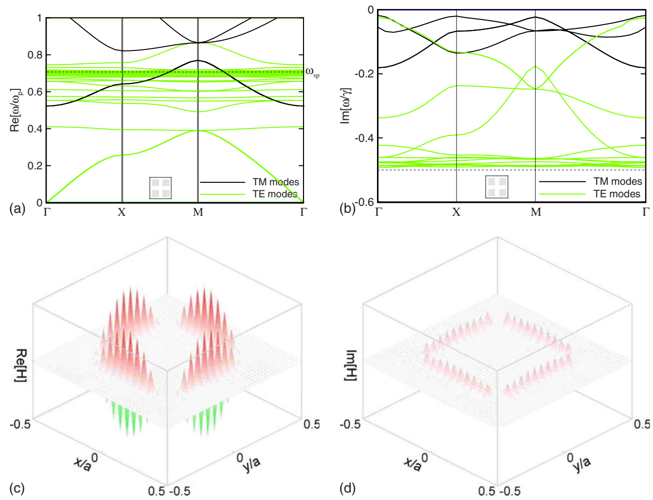

FIG. 2. (Color online) Dispersion relation and mode patterns for a periodic array of square metal columns with width 0.6*a* based on the Drude model with $\omega_p a/2\pi c=1$ and $\gamma/\omega_p=0.2$. (a) Real and (b) imaginary frequency branches; (c) real and (d) imaginary parts of the magnetic field for a typical surface plasmon mode at the point Γ.

highly localized near the interface, as in the case without damping [19]. In addition, the imaginary part of the magnetic field is in phase with the real part, and grows in proportion to the damping constant, as for periodic layered structures. In the present study, the validity of damping effect is limited to the frequency range where the Drude model is appropriate. Outside this range, the interface matching method has to be designed according to specific considerations.

IV. CONCLUDING REMARKS

In conclusion, an interface matching method was proposed for solving surface plasmon modes with damping in plasmonic crystals. By matching the boundary condition at the interface, the original nonlinear eigenvalue problem is reformulated as a cubic eigensystem, which in turn is recast into a linear equation that can be solved by standard algorithms. The damping constant is regarded as a crucial parameter that plays an important role in the properties of surface plasmons, instead of a small perturbation to the case without damping. The effect of damping is manifest on the complex nature of eigenfrequencies and eigenfields. The real frequency branches retain the basic dispersion features of the undamped system, with the surface plasma frequency ω_{sp} slightly attenuated. The imaginary frequency that represents the decay factor in time is proportional to the damping constant γ and bounded by $-\gamma/2$. Meanwhile, the complex eigenfields indicate a phase lag of the field oscillation, compared to the case without damping. These features apply to periodic metal layered structures, where the basic dispersion is described by the acoustical and optical modes, as well as periodic arrays of metal columns, where the dispersion is characterized by intensive surface plasmon modes gathered around ω_{sp}.

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